Calculus Warm Up # 9-4

Set up the integral for arc length then use Simpson's Rule with n = 4 to approximate the integral on [0, π].

$y = \sin x$ (Simpson's Rule: p. 283)

HW Questions: p. 497

1. $\int (3x - 2)^4 \, dx$
3. $\int (-2x + 5)^{3/2} \, dx$
5. $\int \left[v + \frac{1}{(3v - 1)^3} \right] dv$
7. $\int \frac{t^2 - 3}{-t^3 + 9t + 1} \, dt$
9. $\int \frac{x^2}{x - 1} \, dx$
11. $\int \left(\frac{1}{3x - 1} - \frac{1}{3x + 1} \right) dx$

13. $\int t \sin t^2 \, dt$

15. $\int \cos x \, e^{\sin x} \, dx$

17. $\int \frac{(1+e^t)^2}{e^t} \, dt$

19. $\int \sec 3x \tan 3x \, dx$

21. $\int \frac{2}{e^{-x}+1} \, dx$ $\rightarrow$ let $u = e^{-x}+1$

$du = -e^{-x}dx$, not there :(so create it !

23. $\int \frac{1}{1-\cos x} \, dx$

$$2\int \frac{e^x}{e^x(e^{-x}+1)} dx$$

$$= 2\int \frac{e^x}{1+e^x} dx \rightarrow \text{Now try again}$$

$u = 1 + e^x$

$du = e^x dx$:)

$$= 2\int \frac{1}{u} du$$

21. $\int \frac{2}{e^{-x} + 1}\, dx$

23. $\int \frac{1}{1 - \cos x}\, dx \quad \frac{1 + \cos x}{1 + \cos x}\, dx$

$\frac{1 + \cos x}{1 - \cos^2 x}$

$\int \frac{1 + \cos x}{\sin^2 x}\, dx$

$\int \frac{1}{\sin^2 x}\, dx + \int \frac{\cos x}{\sin^2 x}\, dx$ $\qquad u = \sin x$

$du = \cos x\, dx$

$\int \csc^2 x\, dx$

41. $\int (1 + 2x^2)^2\, dx$

43. $\int x\left(1 + \frac{1}{x}\right)^3 dx$

45. $\int (1 + e^x)^2\, dx$

51. $\int_0^1 xe^{-x^2}\,dx$

53. $\int_1^e \frac{1-\ln x}{x}\,dx$

55. $\int_0^4 \frac{2x}{\sqrt{x^2+9}}\,dx$

Chapter 9: Integration Techniques

9.1 Fitting Integrands to the basic formulas

1) Using Algebra
2) Creating du by: add & subtract or
   multiply and divide
3) Recognizing u & du (may be disguised!)
4) Use Trig Identities
5) Multiply & Divide by a conjugate
6) Complete the Square

4) Use Trig Identities

$$\int \tan^2(2x)\,dx$$

$$= \int \left(\sec^2(2x) - 1\right) dx$$

$$= \frac{1}{2}\int 2\sec^2(2x)\,dx - \int 1\,dx$$

$$= \frac{1}{2}\tan(2x) - x + C$$

5) Multiply & Divide by a conjugate

$$\int \frac{1}{1+\sin x}\,dx \quad \cdot \frac{1-\sin x}{1-\sin x}$$

$$= \int \frac{1-\sin x}{1-\sin^2 x}\,dx$$

$$= \int \frac{1}{\cos^2 x}\,dx + \int \frac{-\sin x}{\cos^2 x}\,dx$$

$$= \int \sec^2 x\,dx + \int \frac{1}{u^2}\,du \qquad \begin{aligned} u &= \cos x \\ du &= -\sin x \end{aligned}$$

$$= \tan x + \frac{u^{-1}}{-1} + C$$

$$= \tan x - \frac{1}{\cos x} + C$$

$$\text{or } \tan x - \sec x + C$$

6) Complete the Square

$$\int \frac{1}{x^2+2x+5}\,dx$$

$x^2+2x+\underline{1}+5-\underline{1}$

$(x+1)^2+4$

$$= \int \frac{1}{(x+1)^2+2^2}\,dx$$

$a=2$

$u=(x+1)$

$$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right)+C$$

6) Complete the Square

$$\int \frac{1}{2x^2-8x+14}\,dx$$

$$\int \frac{1}{2\left[(x-2)^2+3\right]}\,dx$$

$2(x^2-4x+\underline{4}+7\underline{-4})$

$2\left[(x-2)^2+3\right]$

$$\frac{1}{2}\int \frac{1}{u^2+a^2}\,du$$

$u=x-2$

$a=\sqrt{3}$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\arctan\left(\frac{x-2}{\sqrt{3}}\right)+C$$

$$= \frac{\sqrt{3}}{6}\arctan\left[\frac{\sqrt{3}(x-2)}{3}\right]+C$$

The derivatives of the six inverse trigonometric functions occur in three pairs. In each pair the derivative of one function is the negative of the other. For example,

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \quad \text{and} \quad \frac{d}{dx}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}.$$

When listing the *antiderivative* that corresponds to each of the inverse trigonometric functions, we need use only one member from each pair. For example, we choose to use $\arcsin x$ as the antiderivative of $1/\sqrt{1-x^2}$, rather than $-\arccos x$. The next theorem gives one antiderivative formula for each of the three pairs.

Let u be a differentiable function of x, and let $a > 0$.

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

Check Answers: Classwork Blue WS

1) $\arcsin\left(\frac{x}{2}\right)+C$

2) $\frac{1}{3\sqrt{2}}\arctan\left(\frac{3x}{\sqrt{2}}\right)+C$

3) $-\sqrt{4-x^2}+2\arcsin\left(\frac{x}{2}\right)+C$

4) $\frac{3x^2}{2}-6\ln(x^2+4)-\arctan\left(\frac{x}{2}\right)+C$

5) $\arcsin(x+1)+C$

6) $\frac{1}{2}\arcsin(t^2)+C$

Check Answers: Classwork Blue WS

7) $\frac{\pi}{18}$

8) $\frac{\pi}{6}$

9) $\frac{x^2}{2} - \frac{1}{2}\ln(x^2+1) + C$

10) $\frac{\pi^2}{32}$

2) $\frac{1}{3}\int \frac{1 \cdot 3}{(\sqrt{2})^2 + (3x)^2}\,dx$ $\quad a = \sqrt{2}$

$u = 3x \quad du = 3\,dx$

$\frac{1}{3} \cdot \frac{1}{\sqrt{2}} \arctan\left(\frac{3x}{\sqrt{2}}\right) + C$

6) $\frac{1}{2}\int \frac{2t}{\sqrt{1-(t^2)^2}}\,dt \quad u = t^2 \quad du = 2t\,dt$

$a = 1$

$\frac{1}{2}\arcsin(t^2) + C$

4) $\int 3x\,dx + \int \frac{-12x-2}{x^2+4}\,dx$

$$\begin{array}{r} 3x \\ x^2+4 \overline{)\,3x^3 \qquad\qquad - 2} \\ -(3x^3+12x) \\ \hline -12x-2 \end{array}$$

$\int 3x\,dx - 6\int \frac{2x}{x^2+4}\,dx - 2\int \frac{1}{x^2+4}\,dx$ $\quad u = x$, $a = 2$

7) $\frac{1}{3}\int \frac{1\cdot 3}{\sqrt{1^2-(3x)^2}}\,dx$ $\quad u = 3x$ $\quad du = 3\,dx$, $a = 1$

$\frac{1}{3}\arcsin(3x)\Big]_0^{1/6}$

$\frac{1}{3}\left(\arcsin\frac{1}{2} - \arcsin 0\right)$

$\frac{\pi}{6}$

9) $\int x\,dx - \int \frac{x}{x^2+1}\,dx$

$$\begin{array}{r} x \\ x^2+1 \overline{\big) x^3} \\ -(x^3+x) \\ \hline -x \end{array}$$

10) $\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

$u = \arcsin x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \int u\,du \ \ldots$

Classwork: Purple WS

Answers

1) $\operatorname{arcsec}|2x| + C$

2) $\arcsin e^x + C$

3) $2\arctan(\sqrt{x}) + C$

4) $\frac{\pi}{2}$

5) $\arcsin\left(\frac{x+2}{2}\right) + C$

6) $\frac{\pi}{4}$

HW: p. 497, # 25 - 39 odd,
47, 49, 57 - 61 odd,
64 - 68 even

☆ Be ready to turn in Blue WS and warm-up tomorrow.